

13 Quadratische Gleichungen

13.1 Übungen

Lösen Sie die Gleichung

Level A

Aufgabe 1.

$$x^2 + 6x + 9 = 0$$

Aufgabe 2.

$$x^2 + 2x - 24 = 0$$

Aufgabe 3.

$$3y^2 = 10 - 29y$$

Aufgabe 4.

$$t^2 - 10t + 37 = 0$$

Lösung:

$$t^2 - 10t + 37 = 0$$

$$D = 10^2 - 4 \cdot 1 \cdot 37 = 100 - 148 = -48$$

Antwort : *korneynet*

Aufgabe 5.

$$2p^2 - 4p - 17 = 0$$

Level B

Aufgabe 1.

$$0,7y^2 - 1,3y - 2 = 0$$

Aufgabe 2.

$$(t + 4)^2 = 3t + 40$$

Aufgabe 3.

$$(2p - 3)^2 = 11p - 19$$

Aufgabe 4.

$$3(f + 4)^2 = 10f + 32$$

Aufgabe 5.

$$(m + 1)^2 = (2m - 1)^2$$

Level C

Aufgabe 1.

$$\frac{x-4}{x-5} + \frac{x-6}{x+5} = 2$$

Aufgabe 2.

$$\frac{t+3}{t-3} + \frac{t-3}{t+3} = 3\frac{1}{3}$$

Aufgabe 3.

$$\frac{1}{2-p} - 1 = \frac{1}{p-2} - \frac{6-p}{3p^2-12}$$

Aufgabe 4.

$$\frac{k+3}{k-3} + \frac{k-3}{k+3} = 3\frac{1}{3}$$

Aufgabe 5.

$$\frac{7f-3}{f-f^2} = \frac{1}{f-1} - \frac{5}{f(f-1)}$$

13.2 Lösung

Level A

Aufgabe 1.

$$x^2 + 6x + 9 = 0$$

Lösung:

$$x^2 + 6x + 9 = 0$$

$$D = 6^2 - 4 * 1 * 9 = 36 - 36 = 0$$

$$x_{1,2} = \frac{-6 \pm 0}{2 * 1} = \frac{-6}{2} = -3$$

Antwort : $x = -3$

Aufgabe 2.

$$x^2 + 2x - 24 = 0$$

Lösung:

$$\begin{cases} x^2 + 2x - 24 = 0 \\ x_1 + x_2 = -2, \\ x_1 * x_2 = -24; \\ x_1 = -6, \\ x_2 = 4; \end{cases}$$

Antwort : $x_1 = -6; x_2 = 4$

Aufgabe 3.

$$3y^2 = 10 - 29y$$

Lösung:

$$3y^2 + 29y - 10 = 0$$

$$D = 29^2 + 4 * 3 * 10 = 841 + 120 = 961 = 31^2$$

$$y_1 = \frac{-29-31}{2*3} = \frac{-60}{6} = -10$$

$$y_2 = \frac{-29+31}{2*3} = \frac{2}{6} = \frac{1}{3}$$

Antwort : $y_1 = -10; y_2 = \frac{1}{3}$

Aufgabe 4.

$$t^2 - 10t + 37 = 0$$

Lösung:

$$t^2 - 10t + 37 = 0$$

$$D = 10^2 - 4 * 1 * 37 = 100 - 148 = -48$$

Antwort : KeineWurzeln

Lösung:

Keine Lösung

Aufgabe 5.

$$2p^2 - 4p - 17 = 0$$

Lösung:

$$2p^2 - 4p - 17 = 0$$

$$D = 4^2 + 4 * 2 * 17 = 16 + 136 = 152$$

$$p_{1,2} = \frac{4 \pm \sqrt{152}}{2 * 2} = \frac{4 \pm \sqrt{152}}{4}$$

$$\text{Antwort : } p_{1,2} = \frac{4 \pm \sqrt{152}}{4}$$

Level B

Aufgabe 1.

$$0,7y^2 - 1,3y - 2 = 0$$

Lösung:

$$0,7y^2 - 1,3y - 2 = 0$$

$$D = (1,3)^2 + 4 * 0,7 * 2 = 1,69 + 5,6 = 7,29 = (2,7)^2$$

$$y_1 = \frac{1,3 - 2,7}{2 * 0,7} = \frac{-1,4}{1,4} = -1$$

$$y_2 = \frac{1,3 + 2,7}{2 * 0,7} = \frac{4}{1,4}$$

$$\text{Antwort : } y_1 = -1; y_2 = \frac{4}{1,4}$$

Aufgabe 2.

$$(t + 4)^2 = 3t + 40$$

Lösung:

$$t^2 + 8t + 16 - 3t - 40 = 0$$

$$t^2 + 5t - 24 = 0$$

$$\left[\begin{array}{l} t_1 + t_2 = -5, \end{array} \right.$$

$$\left[\begin{array}{l} t_1 * t_2 = -24; \end{array} \right.$$

$$\left[\begin{array}{l} t_1 = -8, \end{array} \right.$$

$$\left[\begin{array}{l} t_2 = 3; \end{array} \right.$$

$$\text{Antwort : } t_1 = -8; t_2 = 3$$

Aufgabe 3.

$$(2p - 3)^2 = 11p - 19$$

Lösung:

$$4p^2 - 12p + 9 - 11p + 19 = 0$$

$$4p^2 - 23p + 28 = 0$$

$$D = 23^2 - 4 * 4 * 28 = 529 - 448 = 81 = 9^2$$

$$p_1 = \frac{23-9}{2*4} = \frac{14}{8} = \frac{7}{4}$$

$$p_2 = \frac{23+9}{2*4} = \frac{32}{8} = 4$$

$$\text{Antwort : } p_1 = \frac{7}{4}; p_2 = 4$$

Aufgabe 4.

$$3(f + 4)^2 = 10f + 32$$

Lösung:

$$3(f^2 + 8f + 16) - 10f - 32 = 0$$

$$3f^2 + 24f + 48 - 10f - 32 = 0$$

$$3f^2 + 14f + 16 = 0$$

$$D = 14^2 - 4 * 3 * 16 = 196 - 192 = 4$$

$$f_1 = \frac{-14-2}{6} = -\frac{8}{3}$$

$$f_2 = \frac{-14+2}{6} = -2$$

$$\text{Antwort : } f_1 = -\frac{8}{3}; f_2 = -2$$

Aufgabe 5.

$$(m + 1)^2 = (2m - 1)^2$$

Lösung:

$$m^2 + 2m + 1 = 4m^2 - 4m + 1$$

$$m^2 - 4m^2 + 2m + 4m + 1 - 1 = 0$$

$$-3m^2 + 6m = 0$$

$$-3m(m - 2) = 0$$

$$\left[\begin{array}{l} -3m = 0, \\ m - 2 = 0; \\ m = 0, \\ m = 2; \end{array} \right.$$

Antwort : $m_1 = 0; m_2 = 2$

Level C

Aufgabe 1.

$$\frac{x-4}{x-5} + \frac{x-6}{x+5} = 2$$

Lösung:

$$ODZ : x \neq 5; x \neq -5$$

$$\frac{x-4}{x-5} + \frac{x-6}{x+5} = 2 / * (x-5)(x+5)$$

$$(x-4)(x+5) + (x-6)(x-5) = 2(x+5)(x-5)$$

$$x^2 + 5x - 4x - 20 + x^2 - 5x - 6x + 30 = 2(x^2 - 25)$$

$$2x^2 - 10x + 10 = 2x^2 - 50$$

$$-10x + 60 = 0$$

$$-10x = -60$$

$$x = 6$$

Antwort : $x = 6$

Aufgabe 2.

$$\frac{t+3}{t-3} + \frac{t-3}{t+3} = 3\frac{1}{3}$$

Lösung:

$$ODZ : t \neq 3; t \neq -3$$

$$\frac{t+3}{t-3} + \frac{t-3}{t+3} = 3\frac{1}{3} / * (t-3)(t+3)$$

$$(t+3)(t+3) + (t-3)(t-3) = 3\frac{1}{3} * (t-3)(t+3)$$

$$(t+3)^2 + (t-3)^2 = 3\frac{1}{3} * (t^2 - 9)$$

$$t^2 + 6t + 9 + t^2 - 6t + 9 = \frac{10}{3}t^2 - 30 / * 3$$

$$3t^2 + 3t^2 + 27 + 27 - 10t^2 + 90 = 0$$

$$-4t^2 + 144 = 0 / : 4$$

$$-t^2 + 36 = 0$$

$$t^2 = 36$$

$$t = \pm 6$$

Antwort : $t = \pm 6$

Aufgabe 3.

$$\frac{1}{2-p} - 1 = \frac{1}{p-2} - \frac{6-p}{3p^2-12}$$

Lösung:

$$-\frac{1}{p-2} - 1 = \frac{1}{p-2} - \frac{6-p}{3(p-2)(p+2)} / * 3(p-2)(p+2)$$

$$-3(p+2) - 3(p-2)(p+2) - 3(p+2) + 6 - p = 0$$

$$-3p - 6 - 3p^2 + 12 - 3p - 6 + 6 - p = 0$$

$$-3p^2 - 7p + 6 = 0 / : (-1)$$

$$3p^2 + 7p - 6 = 0$$

$$D = 7^2 + 4 * 3 * 6 = 49 + 72 = 121 = 11^2$$

$$p_1 = \frac{-7-11}{2*3} = \frac{-18}{6} = -3$$

$$p_2 = \frac{-7+11}{2*3} = \frac{4}{6} = \frac{2}{3}$$

Antwort : $p_1 = -3; p_2 = \frac{2}{3}$

Aufgabe 4.

$$\frac{k+3}{k-3} + \frac{k-3}{k+3} = 3\frac{1}{3}$$

Lösung:

$$ODZ : k \neq 3; k \neq -3$$

$$(k+3)^2 + (k-3)^2 = \frac{10}{3} * (k-3)(k+3) / * 3$$

$$3(k+3)^2 + 3(k-3)^2 = 10(k-3)(k+3)$$

$$3(k^2 + 6k + 9) + 3(k^2 - 6k + 9) = 10(k^2 - 9)$$

$$3k^2 + 18k + 27 + 3k^2 - 18k + 27 - 10k^2 + 90 = 0$$

$$-4k^2 + 144 = 0 / : (-4)$$

$$k^2 = 36$$

$$k = \pm 6$$

$$\text{Antwort : } k = \pm 6$$

Aufgabe 5.

$$\frac{7f-3}{f-f^2} = \frac{1}{f-1} - \frac{5}{f(f-1)}$$

Lösung:

$$ODZ : f \neq 0; f \neq 1$$

$$\frac{7f-3}{f-f^2} = \frac{1}{f-1} - \frac{5}{f(f-1)} / * f(f-1)$$

$$-(7f-3) = f-5$$

$$-7f+3-f+5=0$$

$$-8f+8=0$$

$$-8f=-8$$

$$f=1$$

$$\text{Antwort : } -$$

14 Polynomgleichungen

15 Exponentialgleichungen

15.1 Übungen

Lösen Sie die Gleichung

Level A

Aufgabe 1.

$$3^{2x-4} = 729$$

Aufgabe 2.

$$\left(\frac{1}{3}\right)^{2t-3,5} = \frac{1}{\sqrt{3}}$$

Aufgabe 3.

$$5^{f^2+3f} = 5^{3f-8}$$

Aufgabe 4.

$$6^{3x+4} = 6^{x-7}$$

Aufgabe 5.

$$7^{y^2-3y} = 1$$

Level B

Aufgabe 1.

$$\sqrt{2^{-1}} * 2^{x^2-7,5} = 2^{-7}$$

Aufgabe 2.

$$4^y + 2^{y+1} - 24 = 0$$

Aufgabe 3.

$$\frac{0,2^{k-0,5}}{\sqrt{5}} = 5 * 0,04^{k-2}$$

Aufgabe 4.

$$5^{2m-1} + 5^{m+1} = 250$$

Aufgabe 5.

$$3^{2n+1} + 2 * 3^{n+2} = 21$$

Level C

Aufgabe 1.

$$2^{2(x+\sqrt{x^2-2})} - \frac{5}{2} * 2^{x+\sqrt{x^2-2}} = 6$$

Aufgabe 2.

$$\sqrt[y]{64} - \sqrt[y]{2^{3y+3}} + 12 = 0$$

Aufgabe 3.

$$9 * 27^{f-\frac{2}{3}} - \frac{2}{81} * 9^{f+2} = 9$$

Aufgabe 4.

$$5^{2m+1} - 13 * 15^m + 54 * 9^{m-1} = 0$$

Aufgabe 5.

$$(\sqrt{2+\sqrt{3}})^n + (\sqrt{2-\sqrt{3}})^n = 4$$

15.2 Lösung

Level A

Aufgabe 1.

$$3^{2x-4} = 729$$

Lösung:

$$3^{2x-4} = 3^6$$

$$2x - 4 = 6$$

$$2x = 6 + 4$$

$$2x = 10$$

$$x = 5$$

Aufgabe 2.

$$\left(\frac{1}{3}\right)^{2t-3,5} = \frac{1}{\sqrt{3}}$$

Lösung:

$$\left(\frac{1}{3}\right)^{2t-3,5} = \left(\frac{1}{3}\right)^{0,5}$$

$$2t - 3,5 = 0,5$$

$$2t = 0,5 + 3,5$$

$$2t = 4$$

$$t = 2$$

Aufgabe 3.

$$5^{f^2+3f} = 5^{3f-8}$$

Lösung:

$$5^{f^2+3f} = 5^{3f-8}$$

$$f^2 + 3f = 3f - 8$$

$$f^2 + 3f - 3f + 8 = 0$$

$$f^2 + 8 = 0$$

$$f^2 = -8$$

Antwort : kein Wurzel

Aufgabe 4.

$$6^{3x+4} = 6^{x-7}$$

Lösung:

$$6^{3x+4} = 6^{x-7}$$

$$3x + 4 = x - 7$$

$$3x - x + 4 + 7 = 0$$

$$2x + 11 = 0$$

$$2x = -11$$

$$x = -5, 5$$

Aufgabe 5.

$$7y^2 - 3y = 1$$

Lösung:

$$7y^2 - 3y = 7^0$$

$$y^2 - 3y = 0$$

$$y(y - 3) = 0$$

$$\begin{cases} y = 0, \\ y - 3 = 0; \\ y = 0, \\ y = 3. \end{cases}$$

Antwort : $y_1 = 0; y_2 = 3$

Level B

Aufgabe 1.

$$\sqrt{2^{-1}} * 2^{x^2-7,5} = 2^{-7}$$

Lösung:

$$2^{-\frac{1}{2}} * 2^{x^2-7,5} = 2^{-7}$$

$$-\frac{1}{2} + x^2 - 7,5 + 7 = 0 / * 2$$

$$-1 + 2x^2 - 15 + 14 = 0$$

$$2x^2 - 2 = 0$$

$$2x^2 = 2$$

$$x^2 = 1$$

$$x = \pm 1$$

Antwort : $x = \pm 1$

Aufgabe 2.

$$4^y + 2^{y+1} - 24 = 0$$

$$5^{2m-1} + 5^{m+1} = 250$$

Lösung:

$$5^{2m} : 5^1 + 5^m * 5^1 = 250$$

$$\text{Angenommen } 5^m = t, t > 0$$

$$\frac{t^2}{5} + 5t - 250 = 0 / * 5$$

$$t^2 + 25t - 1250 = 0$$

$$D = 25^2 + 4 * 1 * 1250 = 5625 = 75^2$$

$$t_1 = \frac{-25-75}{2*1} = \frac{-100}{2} = -50 (\notin t)$$

$$t_2 = \frac{-25+75}{2*1} = \frac{50}{2} = 25$$

$$5^m = 25$$

$$5^m = 5^2$$

$$m = 2$$

$$\text{Antwort : } m = 2$$

Aufgabe 5.

$$3^{2n+1} + 2 * 3^{n+2} = 21$$

Lösung:

$$3^{2n} * 3^1 + 2 * 3^n * 3^2 = 21$$

$$\text{Angenommen } 3^n = t, t > 0$$

$$3t^2 + 18t - 21 = 0 / : 3$$

$$t^2 + 6t - 7 = 0$$

$$\left[\begin{array}{l} t_1 + t_2 = -6, \\ t_1 * t_2 = -7; \end{array} \right.$$

$$\left[\begin{array}{l} t_1 = -7 (\notin t), \\ t_2 = 1; \end{array} \right.$$

$$3^n = 1$$

$$3^n = 3^0$$

$$n = 0$$

Antwort : $n = 0$

Level C

Aufgabe 1.

$$2^{2(x+\sqrt{x^2-2})} - \frac{5}{2} * 2^{x+\sqrt{x^2-2}} = 6$$

Lösung:

$$\text{Sei } 2^{x+\sqrt{x^2-2}} = t, t > 0$$

$$t^2 - \frac{5}{2}t - 6 = 0 / * 2$$

$$2t^2 - 5t - 12 = 0$$

$$D = 5^2 + 4 * 2 * 12 = 25 + 96 = 121 = 11^2$$

$$t_1 = \frac{5-11}{2*2} = \frac{-6}{4} = -1,5 (\notin t)$$

$$t_2 = \frac{5+11}{2*2} = \frac{16}{4} = 4$$

$$2^{x+\sqrt{x^2-2}} = 4$$

$$2^{x+\sqrt{x^2-2}} = 2^2$$

$$x + \sqrt{x^2-2} = 2$$

$$\sqrt{x^2-2} = 2 - x$$

$$(\sqrt{x^2-2})^2 = (2-x)^2$$

$$x^2 - 2 = 4 - 4x + x^2$$

$$x^2 - 2 - 4 + 4x - x^2 = 0$$

$$4x - 6 = 0$$

$$4x = 6$$

$$x = 1,5$$

Antwort : $x = 1,5$

Aufgabe 2.

$$\sqrt[y]{64} - \sqrt[y]{2^{3y+3}} + 12 = 0$$

Lösung:

$$2^{\frac{6}{y}} - 2^{\frac{3y+3}{y}} + 12 = 0$$

$$2^{\frac{6}{y}} - 2^{3+\frac{3}{y}} + 12 = 0$$

$$2^{\frac{6}{y}} - 8 * 2^{\frac{3}{y}} + 12 = 0$$

$$\text{Sei } 2^{\frac{3}{y}} = t, t > 0$$

$$t^2 - 8t + 12 = 0$$

$$\left[\begin{array}{l} t_1 + t_2 = 8, \end{array} \right.$$

$$\left[\begin{array}{l} t_1 * t_2 = 12; \end{array} \right.$$

$$\left[\begin{array}{l} t_1 = 2, \end{array} \right.$$

$$\left[\begin{array}{l} t_2 = 6; \end{array} \right.$$

$$\left[\begin{array}{l} 2^{\frac{3}{y}} = 2, \end{array} \right.$$

$$\left[\begin{array}{l} 2^{\frac{3}{y}} = 6; \end{array} \right.$$

$$\left[\begin{array}{l} \frac{3}{y} = 1, \end{array} \right.$$

$$\left[\begin{array}{l} \frac{3}{y} = \log_2 6; \end{array} \right.$$

$$\left[\begin{array}{l} y = 3, \end{array} \right.$$

$$\left[\begin{array}{l} y = \frac{3}{\log_2 6}; \end{array} \right.$$

$$\text{Antwort : } y_1 = 3; y_2 = \frac{3}{\log_2 6}$$

Aufgabe 3.

$$9 * 27^{f-\frac{2}{3}} - \frac{2}{81} * 9^{f+2} = 9$$

Lösung:

$$9 * \frac{27^f}{27^{\frac{2}{3}}} - \frac{2}{81} * 9^f * 9^2 = 9$$

$$9 * \frac{27^f}{9} - 2 * 9^f = 9$$

$$27^f - 2 * 9^f = 9$$

$$Pust 3^f = t, t > 0$$

$$t^3 - 2t^2 - 9 = 0t^3 - 3t^2 + t^2 - 9 = 0$$

$$t^2(t-3) + (t-3)(t+3) = 0$$

$$(t-3)(t^2+t+3) = 0$$

$$\left[\begin{array}{l} t = 3, \\ t^2 + t + 3 = 0 (t \notin \mathbb{R}); \end{array} \right.$$

$$3^f = 3$$

$$f = 1$$

$$\text{Antwort : } f = 1$$

Aufgabe 4.

$$5^{2m+1} - 13 * 15^m + 54 * 9^{m-1} = 0$$

Lösung:

$$5^{2m} * 5^1 - 13 * 3^m * 5^m + 54 * \frac{9^m}{9} = 0$$

$$5^{2m} * 5^1 - 13 * 3^m * 5^m + 6 * 9^m = 0 / : 9^m$$

$$\frac{5 * 5^{2m}}{9^m} - \frac{13 * 15^m}{9^m} + \frac{6 * 9^m}{9^m} = 0$$

$$5\left(\frac{5}{3}\right)^{2m} - 13\left(\frac{5}{3}\right)^m + 6 = 0$$

$$Pust \left(\frac{5}{3}\right)^m = t, t > 0$$

$$5t^2 - 13t + 6 = 0$$

$$D = 13^2 - 4 * 5 * 6 = 169 - 120 = 49 = 7^2$$

$$t_1 = \frac{13-7}{2*5} = \frac{6}{10} = \frac{3}{5}$$

$$t_2 = \frac{13+7}{2*5} = \frac{20}{10} = 2$$

$$\left[\left(\frac{5}{3}\right)^m = \frac{3}{5}, \right.$$

$$\left[\left(\frac{5}{3}\right)^m = 2; \right.$$

$$\left[m = -1, \right.$$

$$\left[m = \log_{\frac{5}{3}} 2. \right.$$

$$\text{Antwort : } m_1 = -1; m_2 = \log_{\frac{5}{3}} 2$$

Aufgabe 5.

$$(\sqrt{2 + \sqrt{3}})^n + (\sqrt{2 - \sqrt{3}})^n = 4$$

Lösung:

16 Logarithmusgleichungen

16.1 Übungen

Lösen Sie die Gleichung

Level A

Aufgabe 1.

$$\log_2(4 - a) = 7$$

Aufgabe 2.

$$\log_5(4 + y) = 2$$

Aufgabe 3.

$$\log_7(15 + x) = \log_7 3$$

Aufgabe 4.

$$\log_{\frac{1}{7}}(7 - m) = -2$$

Aufgabe 5.

$$\log_5(7 - x) = \log_5 5(3 - x)$$

Level B

Aufgabe 1.

$$\log_{\frac{1}{7}}(x^2 + x - 5) = -1$$

Aufgabe 2.

$$3\log_2 \frac{1}{2} - \log_2 \frac{1}{32} = \log_2 y$$